Starter

A curve has equation $y = 7 - 2x^5$.

(a) Find $\frac{dy}{dx}$.

(b) Find an equation for the tangent to the curve at the point where $x = 1$.

(c) Determine whether y is increasing or decreasing when $x = -2$.

$$\frac{dy}{dx} = -10x^4$$

When $x = 1$, gradient $= -10$

Tangent is

$y - 5 = -10(x - 1)$ *or* $y + 10x = 15$ etc

When $x = -2$ $\quad \frac{dy}{dx} = -160$ (or < 0)

($\frac{dy}{dx} < 0$ hence) y is **decreasing**

F Exponentials and logarithms

F2

Know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications.

Notes

At AS, students should **know** that the gradient of Ae^{kx} is proportional to the value of the function.

At AS, students are **not** expected to differentiate functions involving e^{kx}

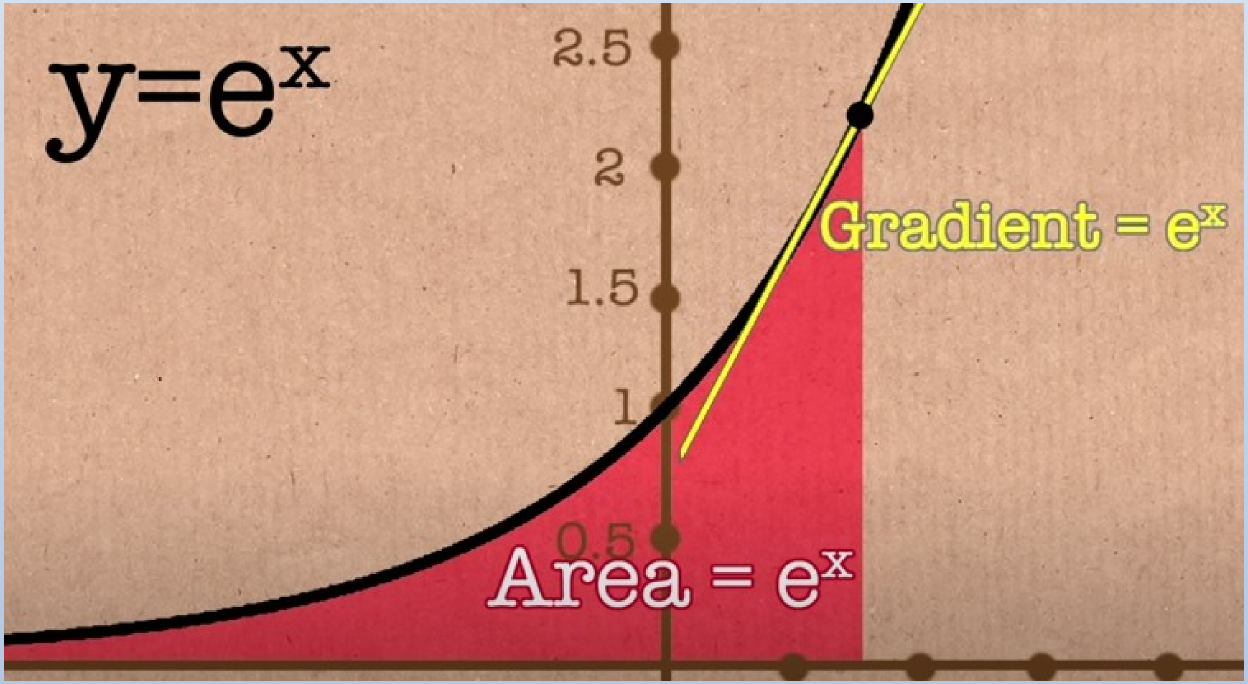
This is an unusual situation where the gradient is expected to be known without differentiation being formally used.

Students should understand that the exponential model is suitable in many applications because, if $y = e^{kx}$, $\frac{dy}{dx} = ky$ i.e. the rate of change of y with respect to x is proportional to y.

5.2 Exponential Functions

The gradient of $y = e^x$

If then



5.2 Exponential Functions

The gradient of $y = e^x$

is a stretch in the -direction with scale factor ½

if the graph is **half** as wide, the gradient is **twice** as steep!

5.2 Exponential Functions

The gradient of $y = e^x$

is a stretch in the -
direction with scale factor 2

if the graph is **twice** as wide,
the gradient is **half** as steep!

In general, for ,

5.2 Exponential Functions

Example 1

Differentiate the following with respect to :

$$\frac{dy}{dx} = 4e^{4x}$$

$$\frac{dy}{dx} = -e^{-x}$$

$$\frac{dy}{dx} = \frac{1}{3}e^{\frac{1}{3}x}$$

$$\frac{dy}{dx} = 6e^{2x}$$

5.2 Exponential Functions

Example 2

Find the gradient of the curve at the points and . Leave your answers as exact solutions.

$$f'(x)=40e^{8x}$$

$$f'(0)=40e^{0}=40$$

$$f'(2)=40e^{16}$$

5.2 Exponential Functions

Example 3a

Write down the gradient of the graph ofat the point and hence find the equation of the tangent.

$$\frac{dy}{dx} = e^x$$

At ,

$$\therefore m = e^3$$

$$\left(x, y\right) \rightarrow \left(3, e^3\right)$$

5.2 Exponential Functions

Example 3b

Find the equation of the normal to at

$$\left(x , y \right) \rightarrow \left(3 , e^3 \right)$$

5.2 Exponential Functions

You try

Determine the equation of the tangent to the curve $y = e^x$ at the point where

a $x = 3$ **b** $x = \frac{1}{2}$

a $y = e^3x - 2e^3$

b $y = e^{\frac{1}{2}}x + \frac{1}{2}e^{\frac{1}{2}}$

$$y = \sqrt{e}\left(x + \frac{1}{2}\right)$$

5.2 Exponential Functions

Example 4

The tangent to the curve $y = e^x$ at point P, where $x = 2$, intersects the y-axis at point Q
Find point Q

The gradient of $y = e^x$ at $P(2, e^2)$ is e^2

At P, the equation of the tangent is $\frac{y - e^2}{x - 2} = e^2$

giving $y = e^2 x - e^2$

When $x = 0$ $y = 0 - e^2 = -e^2$

The point Q is $(0, -e^2)$

The gradient of $y = e^x$ is e^x

Use the equation of a straight line to find the equation of the tangent.

The tangent intersects the y-axis at Q, so $x = 0$

5.2 Exponential Functions

Example 5

Find the exact coordinates and nature of any stationary points on the curve

Hence there is a stationary point at which is a minimum.

5.2 Exponential Functions

Example 6

Worksheet Question

A model for the growth of a colony of bacteria is $P = 500e^{\frac{1}{8}t}$, where P is the number of bacteria after t minutes.

What is the initial rate of growth of the bacteria?

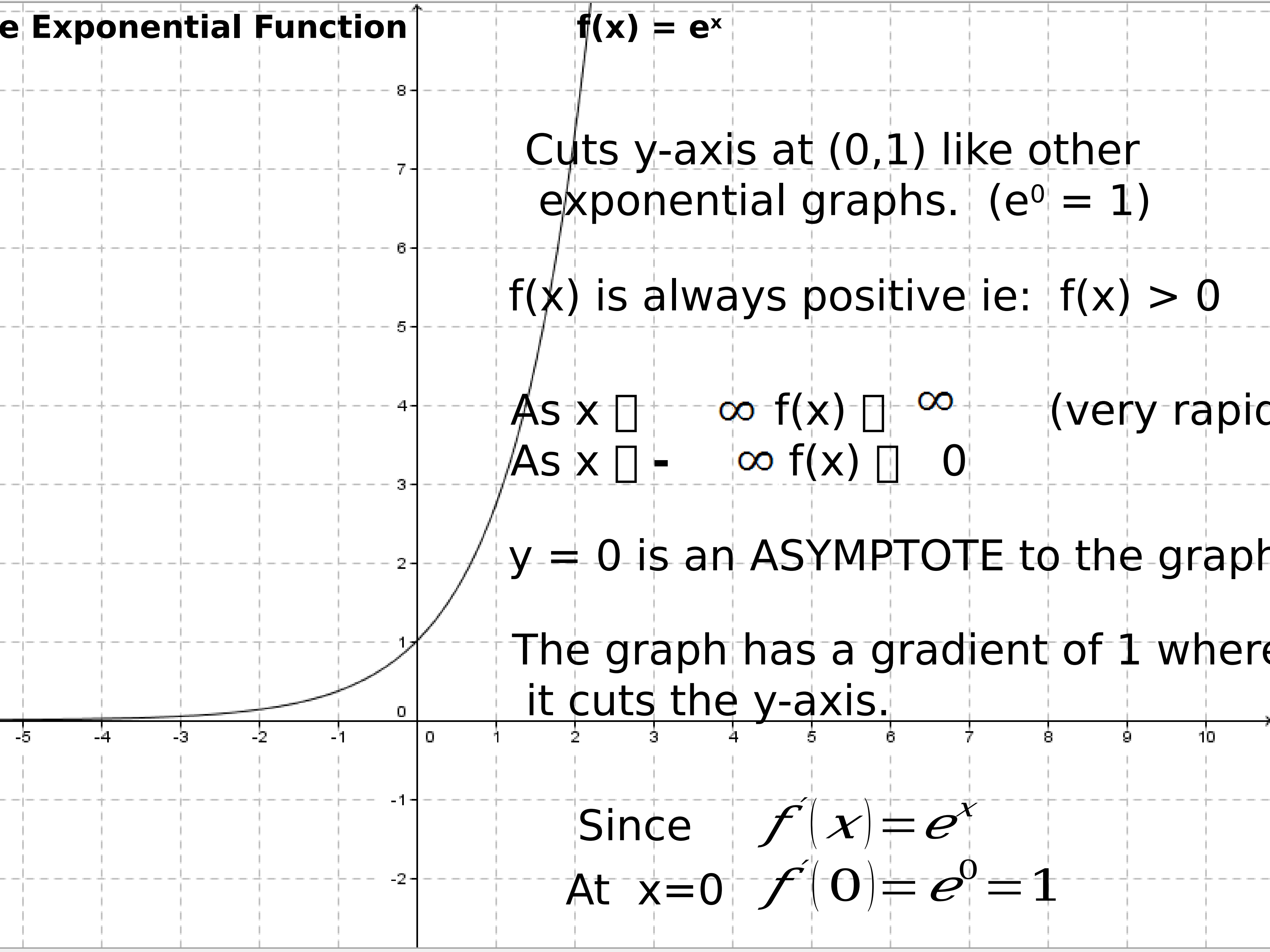
$$\frac{dP}{dt} = \frac{500}{8} e^{\frac{1}{8}t} = \frac{125}{2} e^{\frac{1}{8}t}$$

At ,

bacteria per minute

Extension: can you prove the derivative of from first principles?

The Exponential Function $f(x) = e^x$



Cuts y-axis at (0,1) like other exponential graphs. ($e^0 = 1$)

f(x) is always positive ie: $f(x) > 0$

As $x \to \infty$ $f(x) \to \infty$ (very rapid

As $x \to -\infty$ $f(x) \to 0$

y = 0 is an ASYMPTOTE to the graph

The graph has a gradient of 1 where it cuts the y-axis.

Since $f'(x) = e^x$

At x=0 $f'(0) = e^0 = 1$